

ESTIN

Pattern recognition for image analysis

S5 (DS-AI)

2024-2025

TD° 02

Part1

Exercise 1

You are required to identify the class of a single leaf in an image. All you know is that this image comes from a database containing 200 laurel leaves and 120 olive leaves.

Exercise 2

Suppose that x is a scalar and that a two-category classifier is to be constructed. Under ω_1 , x is Gaussian with mean m_1 and variance σ^2 , and Under ω_2 , x is Gaussian with mean m_2 and variance σ^2 . Show that the decision rule for this case is to compare x to a constant x_0 . Find the value of x_0 in terms of the parameters of this problem.

Exercise 3

Consider the following decision rule for a two-category decision problem where the feature is a scalar x : Given a constant (threshold value x_0), decide ω_1 is the true state of nature if $x > x_0$, otherwise decide ω_2 is the true state of nature.

- Show that the probability of error for this rule is given by

$$P(\text{error}) = P(\omega_1) \int_{-\infty}^{x_0} p(x/\omega_1) dx + P(\omega_2) \int_{x_0}^{+\infty} p(x/\omega_2) dx$$

- By differentiating the expression above, show that a necessary condition to minimize $P(\text{error})$ is that satisfy:

$$p(x_0/\omega_1)P(\omega_1) = p(x_0/\omega_2)P(\omega_2)$$

- What optimality criterion does this condition imply? (Bayes optimal, Maximum Likelihood, etc.)

The pdfs are Gaussians with variance $\sigma^2 = 1/2$ for both classes and mean values 0 and 1, respectively, that is,

$$p(x/\omega_1) = \frac{1}{\sqrt{\pi}} \exp(-x^2)$$

$$p(x/\omega_2) = \frac{1}{\sqrt{\pi}} \exp(-(x-1)^2)$$

- If $P(\omega_1) = P(\omega_2) = 1/2$, compute the threshold value x_0 for minimum error probability.

Exercise 4

Let x be a one-dimensional feature. Suppose that x is to be used to decide between two states of nature ω_1 and ω_2 . $p(x/\omega_1)$ is Gaussian with mean 0 and variance 3; $p(x/\omega_2)$ is Gaussian with mean 2 and variance 1.

1. Sketch the two densities on the same graph as a function of x .
2. What is the likelihood ratio?
3. Suppose that $P(\omega_1) = P(\omega_2) = 1/2$. $\lambda_{11} = \lambda_{22} = 0$. $\lambda_{12} = 1$, $\lambda_{21} = \sqrt{3}$. Find the Bayes decision rule. What is the probability of error of this decision rule?

Exercise 5

In many pattern classification problems one has the option either to assign the pattern to one of N_c classes, or to reject it as being unrecognizable. Let α_i , $i = 1, \dots, N_c$ be the action of deciding that the true state of nature is ω_i , and let α_{N_c+1} be the action of rejecting the pattern as unrecognizable. Let the loss function be

$$\lambda(\alpha_i/\omega_j) = \begin{cases} 0, & i = j \\ \lambda_r, & i = N_c + 1 \\ \lambda_s, & \text{otherwise} \end{cases}$$

Where λ_r and λ_s are non-negative constants; λ_r is the loss associated with rejection, and λ_s is the loss associated with a wrong classification. Show that the minimum risk decision rule is to decide ω_i if $P(\omega_i/\mathbf{x}) > P(\omega_j/\mathbf{x})$ for all j and for

$P(\omega_i/\mathbf{x}) \geq 1 - \lambda_r/\lambda_s$, and reject otherwise. What happens if $\lambda_r = 0$? What happens if

$\lambda_r > \lambda_s$?